Critical Thinking in Hydrogeologic Test Interpretation
Additional materials #2

Basic concepts in groundwater hydraulics

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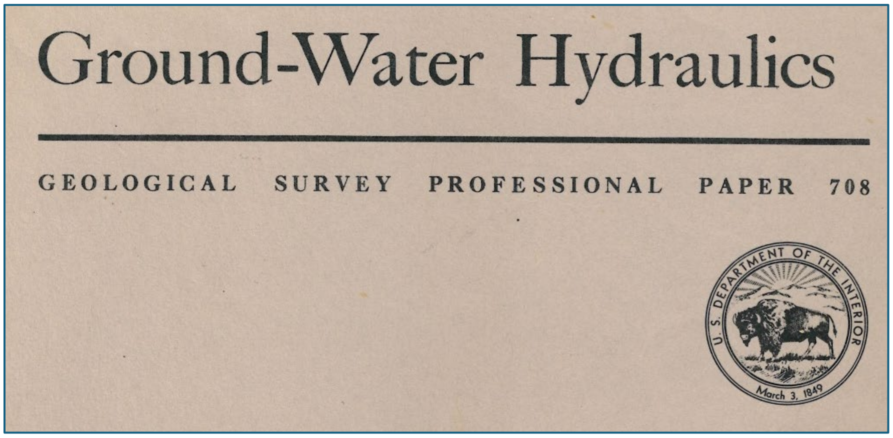
1. Some excellent references for pumping test interpreters

Two excellent starting points for the study of groundwater hydraulics are Lohman (1972) and Kruseman and de Ridder (1990). Both references are available as free downloads.

https://pubs.usgs.gov/publication/pp708



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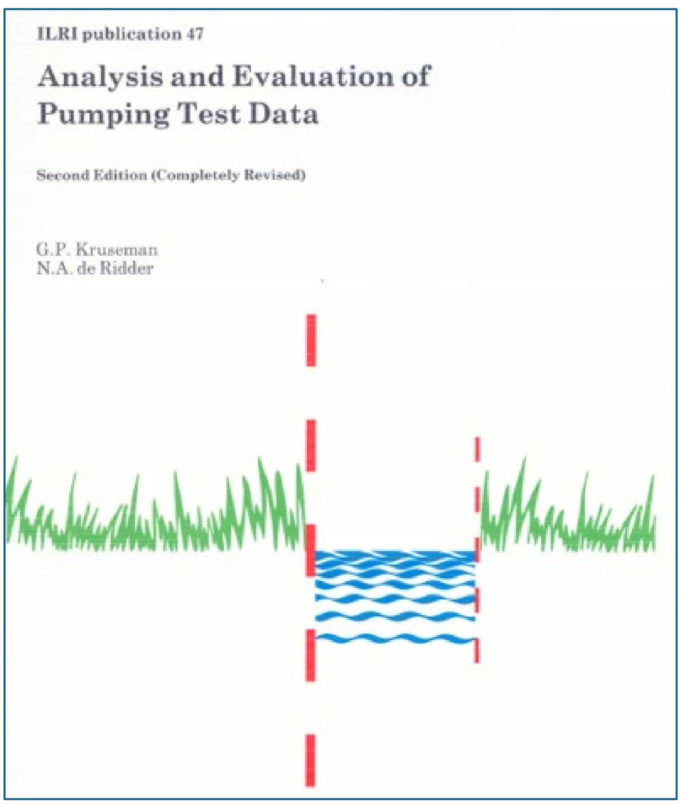
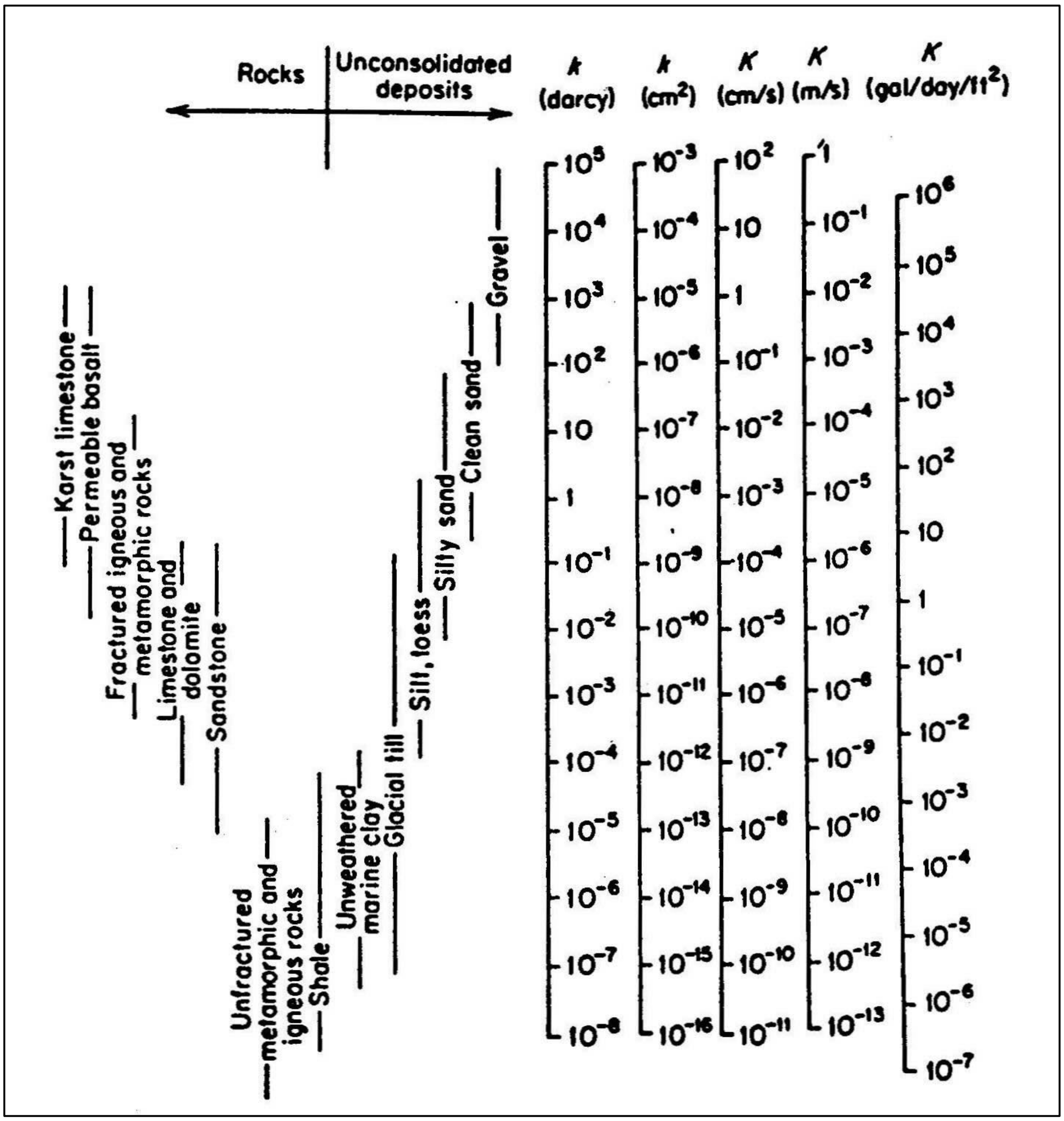


2. Hydraulic conductivity

Hydraulic conductivity (*K*) is the measure of the ease with which water can flow through a porous material (soil or rock). It is a property of the porous medium and the water (specifically the density and dynamic viscosity). A high hydraulic conductivity means water flows easily through the porous medium, while a low value means it does not readily flow through the porous medium. Mathematically, *K* is the proportionality constant in Darcy's Law, linking the flux (flow rate per unit area) and the hydraulic gradient ($q=K\times\delta h/\delta l$). A useful chart for assessing the relative magnitudes of hydraulic conductivities of porous media is reproduced from Freeze and Cherry (1979).



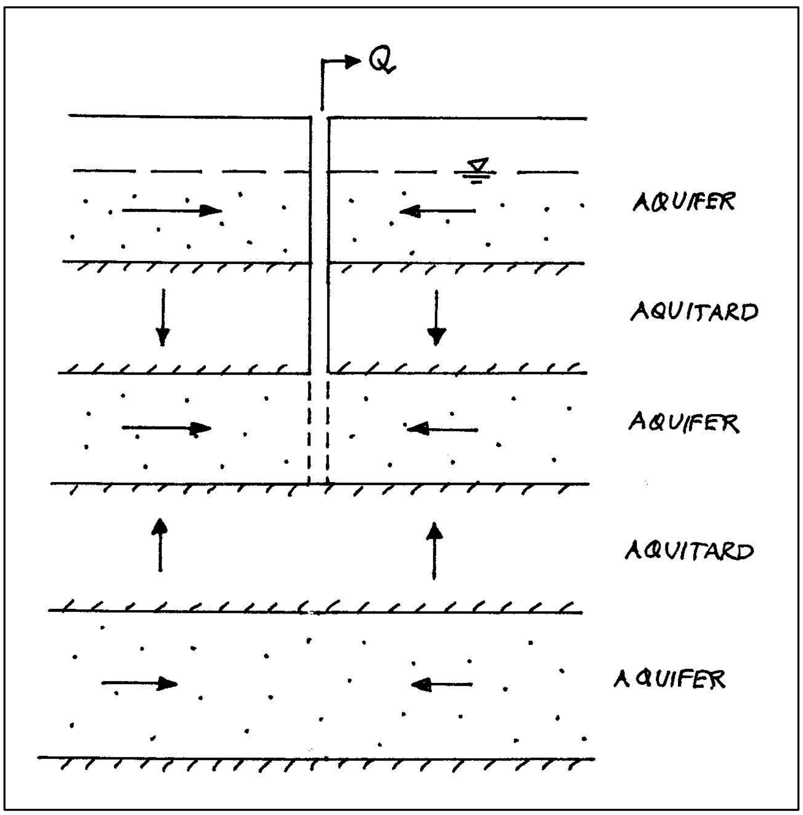
3. Aquifers and aquitards

Operational definitions ...

- An aquifer is a saturated, permeable geologic unit that can transmit significant quantities of water under ordinary hydraulic gradients. The qualifier "significant quantities" may be interpreted as "sufficient to yield economic quantities of water to wells."

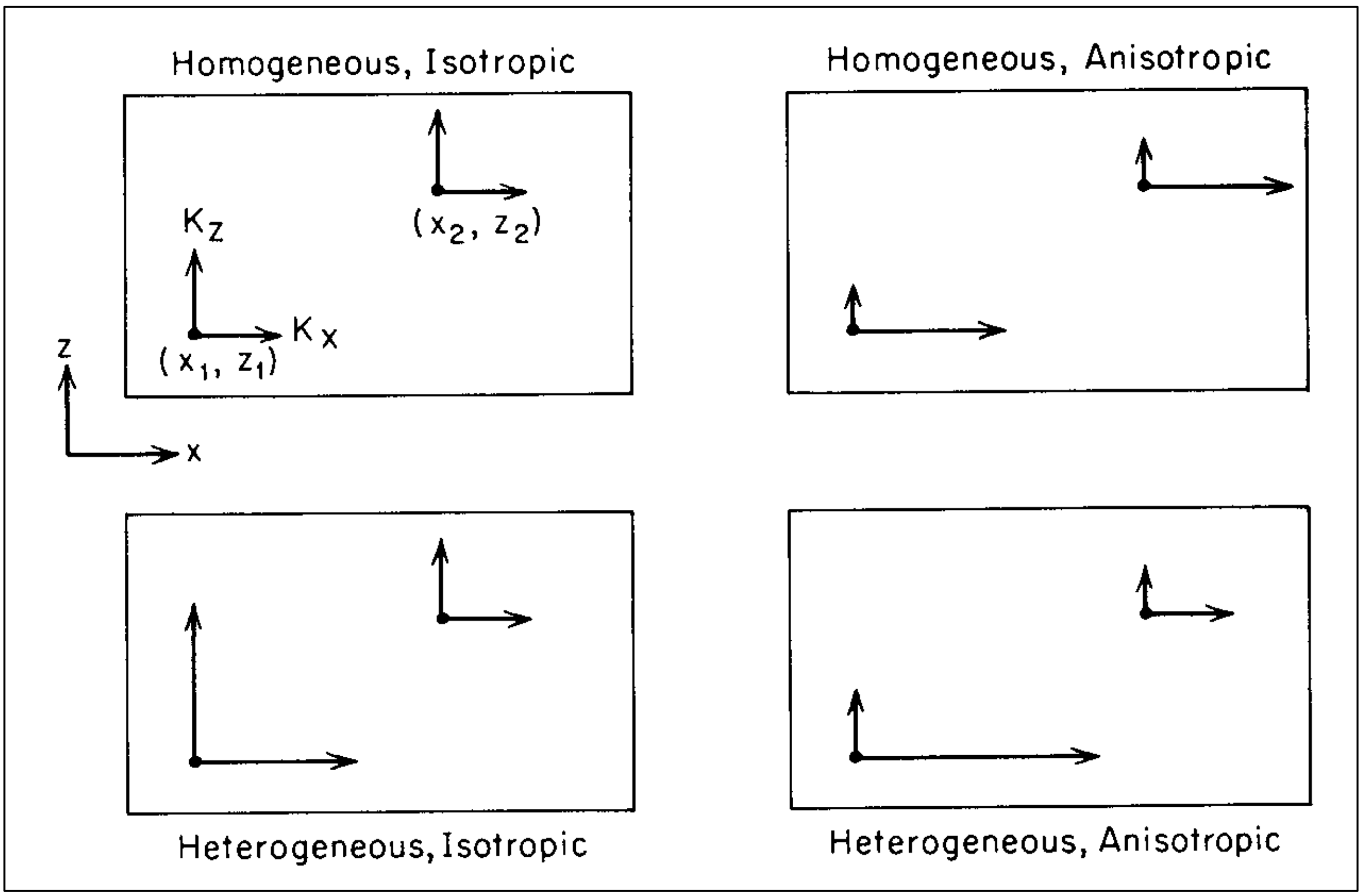
- An aquitard consists of beds that may be sufficiently permeable to transmit water in quantities that are significant with respect to regional groundwater flow, but not sufficiently permeable to support the completion of production wells within them.

In sequences of aquifers and aquitards, groundwater flow is primarily horizontal within aquifers and vertical across aquitards. This simplification is valid for contrasts in hydraulic conductivity greater than 100 (i.e., $K_{\text{aquitard}} < K_{\text{aquifer}}/100$).



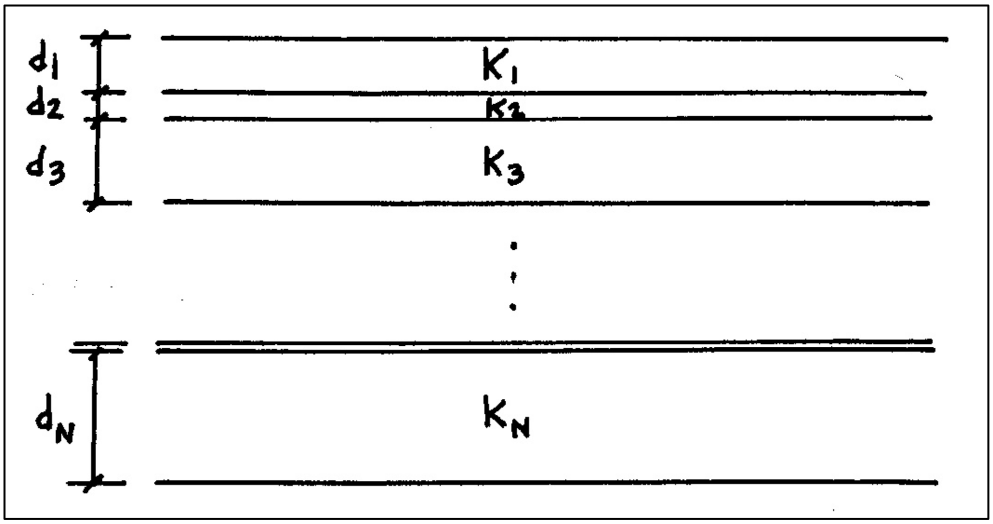
4. Anisotropy and heterogeneity

- If the hydraulic conductivity is *independent* of position within a geologic formation, the formation is *homogeneous*.
- If the hydraulic conductivity depends on position within a geologic formation, the formation is *heterogeneous*.
- If the hydraulic conductivity is *independent* of the direction of measurement in a geologic formation, the formation is *isotropic* at that point.
- If the hydraulic conductivity *varies* with the direction of measurement in a geologic formation, the formation is *anisotropic* at that point.



5. Effective hydraulic conductivities of layered porous media

At the scale of aquifer units, sediments and rocks are generally *anisotropic*. That is, the bulk horizontal and vertical hydraulic conductivities are different. Anisotropy is a manifestation of large-scale anisotropy. To illustrate this, we consider the conceptual model of a perfectly stratified porous medium shown schematically below. Each layer *i* has a uniform, isotropic hydraulic conductivity K_i.



The effective hydraulic conductivities parallel and perpendicular to layering are given by (see Freeze and Cherry, 1979; p. 34, for example):

$$\bar{K}_H = \frac{\sum_{i=1}^{N} K_i d_i}{\sum_{i=1}^{N} d_i}$$

$$\bar{K}_V = \frac{\sum_{i=1}^{N} d_i}{\sum_{i=1}^{N} \frac{d_i}{K_i}}$$

For beds of equal thickness, the effective horizontal hydraulic conductivity is the *arithmetic mean K*, while the effective vertical hydraulic conductivity is the *harmonic mean K*.

$$\bar{K}_H = \frac{1}{N}\sum_{i=1}^{N} K_i$$

$$\bar{K}_V = \frac{1}{\frac{1}{N}\sum_{i=1}^{N} \frac{1}{K_i}}$$

Inspection of the equations for the bulk-average horizontal and vertical hydraulic conductivities reveals that:

- The bulk-average horizontal hydraulic conductivity is dominated by the <u>most</u> permeable stratum; and

- The bulk-average vertical hydraulic conductivity is dominated by the <u>least</u> permeable stratum.

Therefore, we expect values of the anisotropy ratio to generally be less than 1.0:

$$\frac{\bar{K}_V}{\bar{K}_H} < 1.0$$

As a "rule-of-thumb", an anisotropy ratio of 1/10 seems to be assumed frequently in groundwater analyses. A review of the literature suggests that a "blanket" value of 1/10 may not always be appropriate. For heterogeneous materials, significantly larger values on the order of 1/100 to 1/1000 are likely (Freeze and Cherry, 1979; page 34).

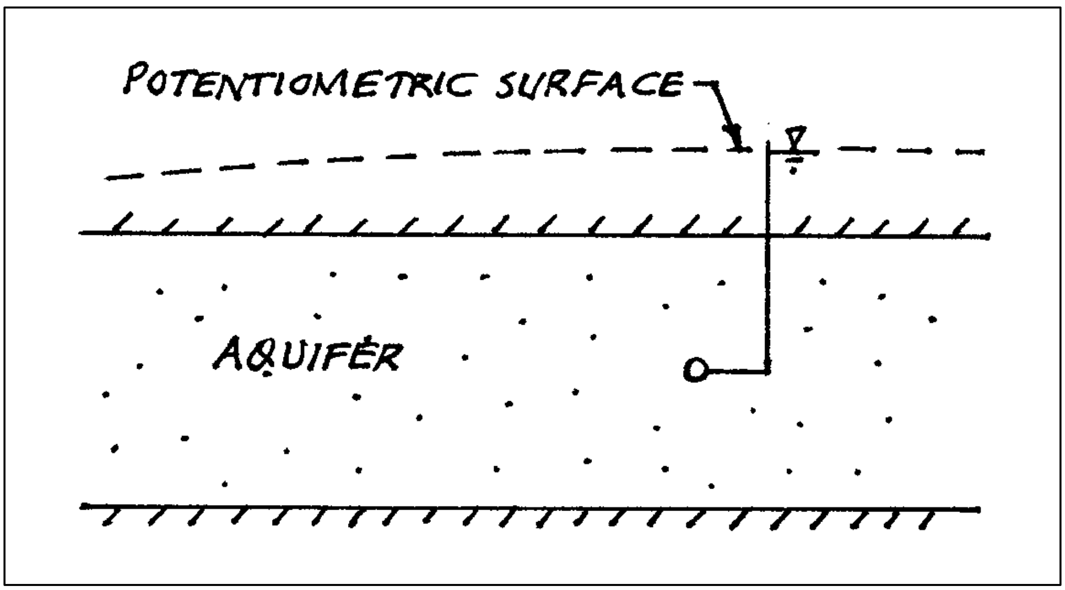
6. Aquifer types

There are three general types of aquifers: confined, leaky and unconfined. These aquifer types are shown schematically below.

Confined aquifer

A confined aquifer is overlain and underlain by relatively impermeable strata. The potentiometric surface always remains above the top of the aquifer and the aquifer thickness does not change with pumping.

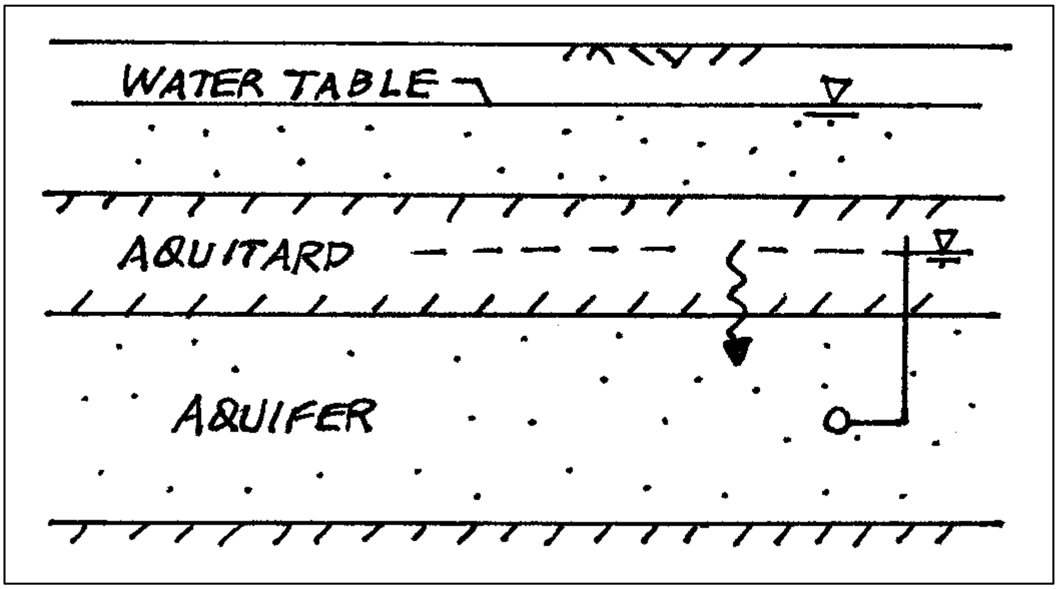
During the period of transient response during a pumping test, the water pumped from the well is derived from compression of the sediments/fractures structure and expansion of the water due to depressurization. These contributions are referred to as “confined storage”.



Leaky aquifer

A leaky aquifer is overlain and underlain by low-permeability strata that contribute water to the aquifer through leakage from over and underlying aquifers. The potentiometric surface in the aquifer always remains above the top of the aquifer and the aquifer thickness does not change with pumping.

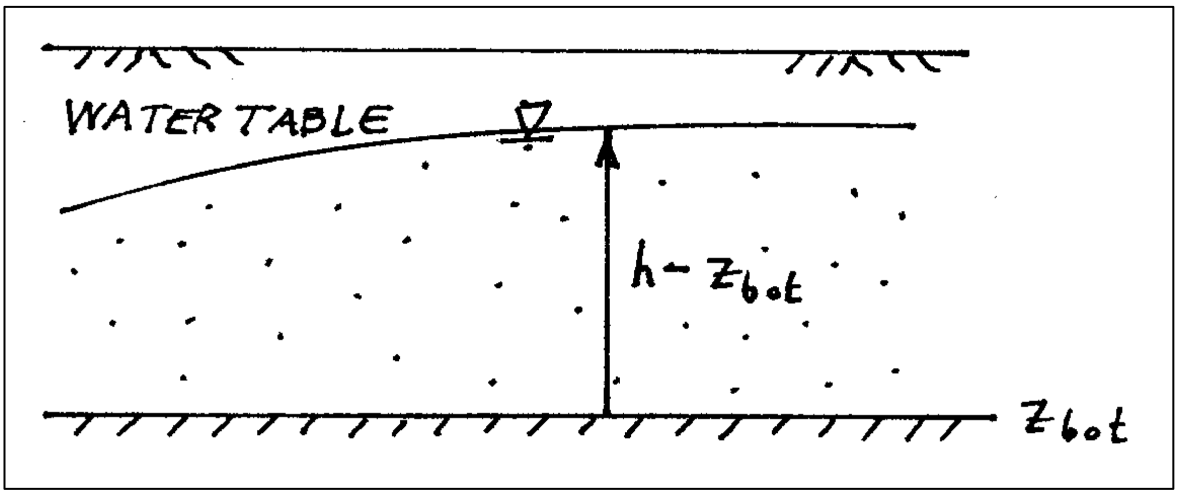
During the initial period of transient response during a pumping test, the water pumped from the well is derived from confined storage. The drawdowns stabilize when all of the pumped water is supplied by leakage across the aquitards.



Unconfined aquifer

The top surface of an unconfined aquifer is the regional water table and the aquifer is underlain by a relatively impermeable stratum. The saturated thickness of the aquifer is the difference between the elevations of the water table and the top of the underlying aquitard. The thickness of the aquifer is the difference between the elevations of the water table and the base of the aquifer. The saturated thickness changes with pumping.

During the earliest period of transient response during a pumping test, the water pumped from the well is derived from confined storage. This period of often too brief to be observed. As pumping continues, the drawdown cone propagates to the water table and the water is supplied by drainage of the capillary fringe. Eventually, the pumped water is derived by drainage of the pores at the water table.



7. Transmissivity

Transmissivity quantifies the rate at which water can be transmitted through an aquifer. The transmissivity is defined as the product of the aquifer horizontal hydraulic conductivity and thickness (*B*).

$$T = K_{\mathrm{H}} \times B$$

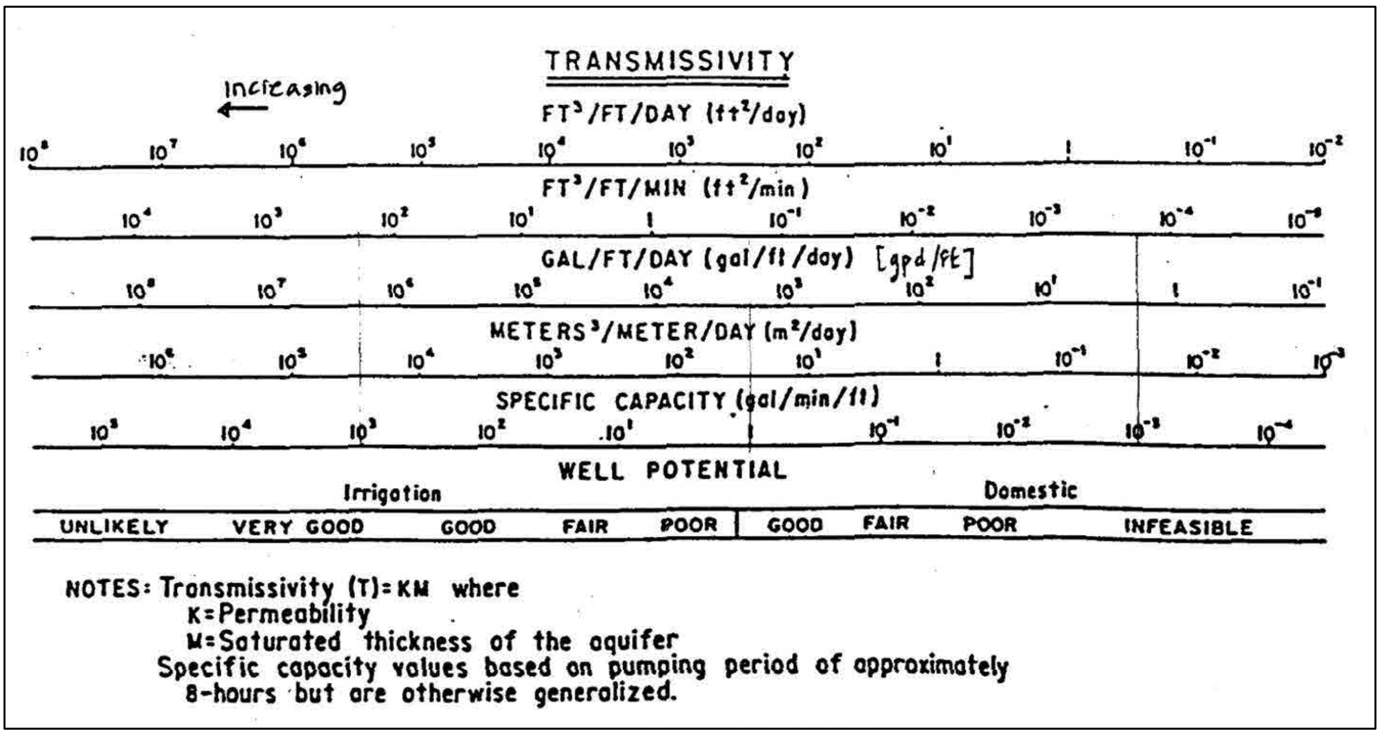
For a horizontal hydraulic gradient $\delta h/\delta l$, the discharge over a length *W* perpendicular to groundwater flow is:

$$Q = T \times W \times \frac{\partial h}{\partial l}$$

For radial flow to a well, the discharge around a circle of radius *r* centered on the well is:

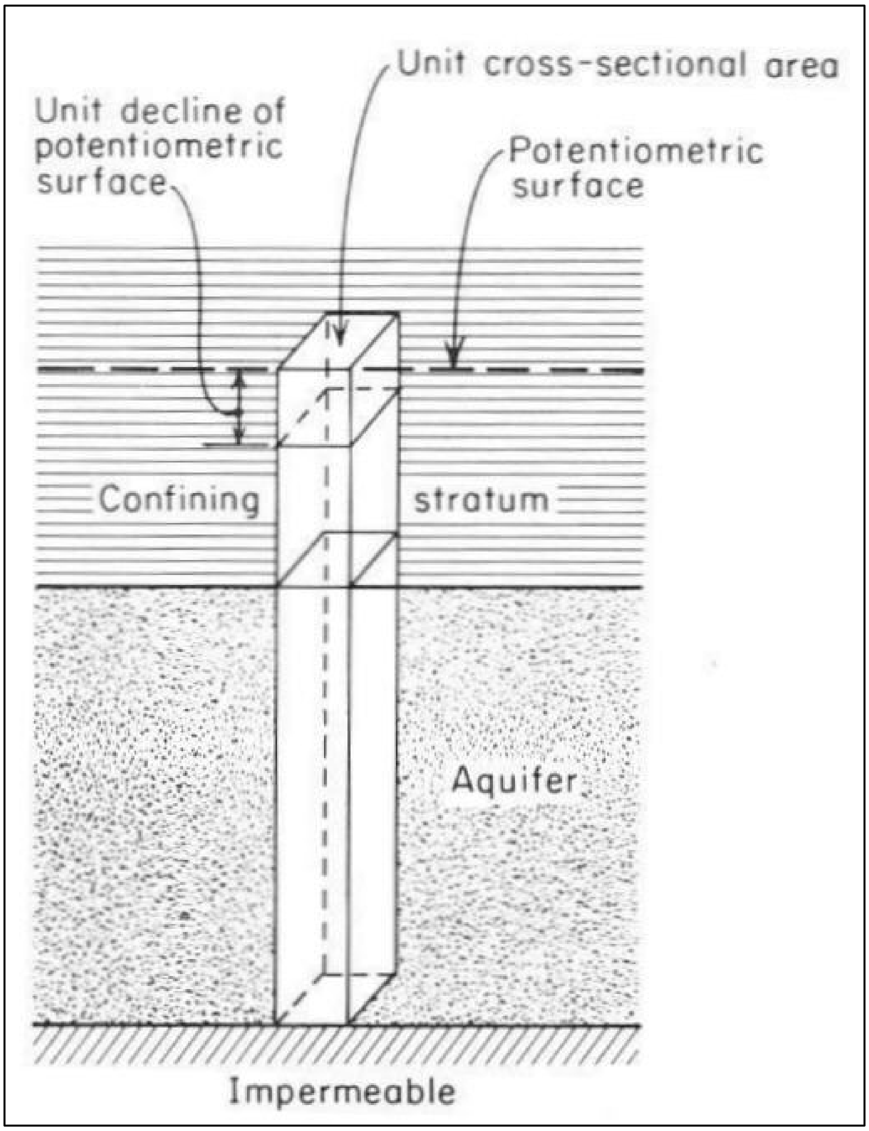
$$Q = T \times 2\pi r \times \frac{\partial h}{\partial r}$$

A useful chart for assessing the magnitude of the transmissivity is reproduced below (U.S. Bureau of Reclamation, 1995; Figure 5-4).



8. Confined storage coefficients

The *specific storage, S_s,* of a saturated aquifer is defined as the volume of water that a unit volume of aquifer releases from storage for a unit decline in hydraulic head (Freeze and Cherry, 1979; p. 58). The specific storage has units of L^{-1}. The *storativity* of a saturated aquifer, also referred to as the *confined storage coefficient*, *S*, is defined as the volume of water that an aquifer releases from storage per unit surface area of aquifer per unit decline in the hydraulic head (Freeze and Cherry, 1979; p. 60). The storativity is given by the product of the specific storage and the aquifer thickness and is dimensionless.



The stored water is released from the compaction (consolidation) of the aquifer and the expansion of water. Jacob (1940; p. 576) derived the following expression for the specific storage:

$$S_s = \rho_w g(\alpha + n\beta)$$

Here ρ_w is the density of water, *g* is the acceleration due to gravity, α is the compressibility of the aquifer, β is the compressibility of water, and *n* is the porosity of the sediments.

<u>SI units</u>:

[ρ] = kg/m^3;
[g] = m/s^2;
[α] = m^2/N (or Pa^{-1}, or m-s^2/kg);
[β] = m^2/N (or Pa^{-1}, or m-s^2/kg); and
[*n*] = dimensionless.

For SI units, the specific storage has units of m^{-1}.

The *storativity* is calculated as the product of the specific storage and the aquifer thickness:

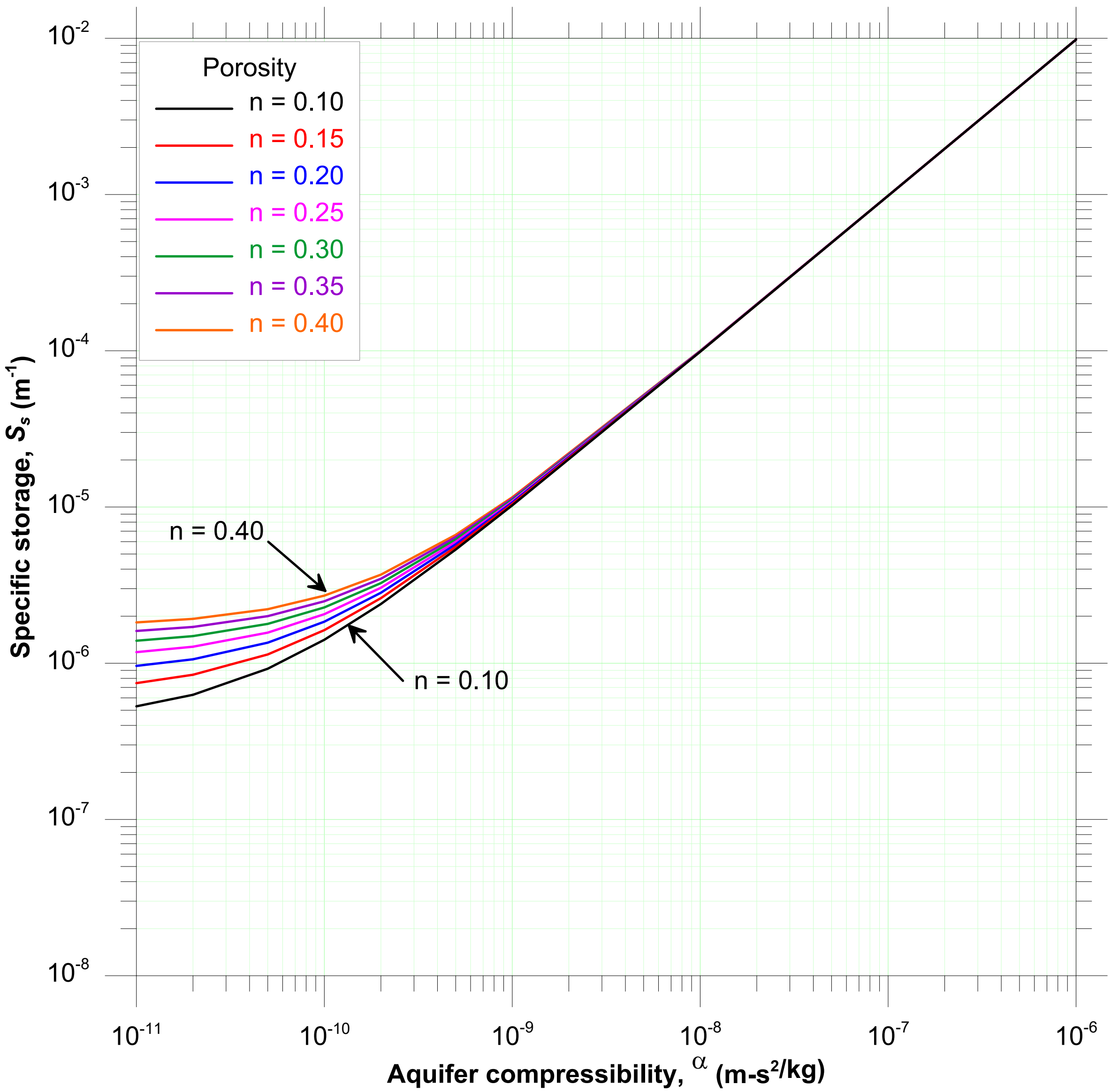
$$\begin{aligned} S &= S_s B \\ &= \rho_w g(\alpha + n\beta)B \end{aligned}$$

Here *B* is the aquifer thickness. The storativity is dimensionless.

Younger (1993) presents "typical" order-of-magnitude values of the compressibility of aquifer materials. These values are reproduced below.

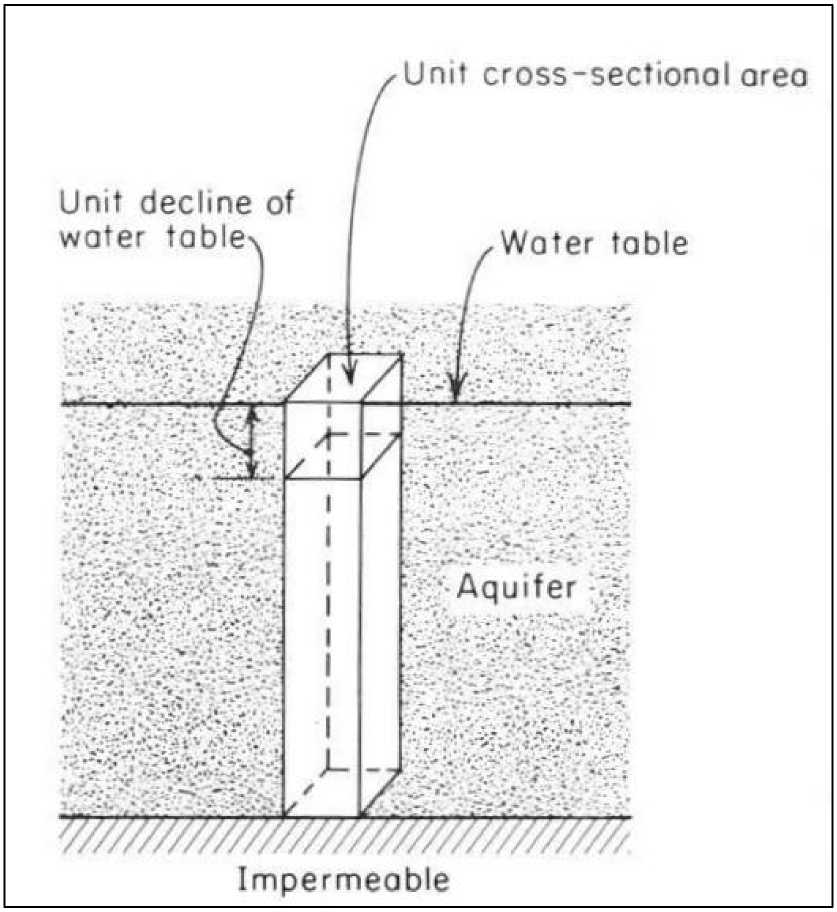
Porous medium	α **(m-s^2/kg)**
Clay	10^{-6}
Silt, fine sand	10^{-7}
Medium sand	10^{-8}
Coarse sand	10^{-9}
Gravel	10^{-10}
Intact rock	10^{-11}

The porosity of sand and gravel aquifers generally ranges from between about 0.10 and 0.40 (Freeze and Cherry, 1979). Typical values for the density and compressibility of water are ρ_w = 1000 kg/m^3 and β = 4.4×10^{-10} m-s^2/kg. Using these values, the specific storage for a range of compressibilities and porosities is plotted below. As shown in the plot, the specific storage depends only weakly on the porosity, and is essentially independent of porosity for aquifer compressibilities greater than 10^{-9} m-s^2/kg.



9. Unconfined storage coefficient (specific yield)

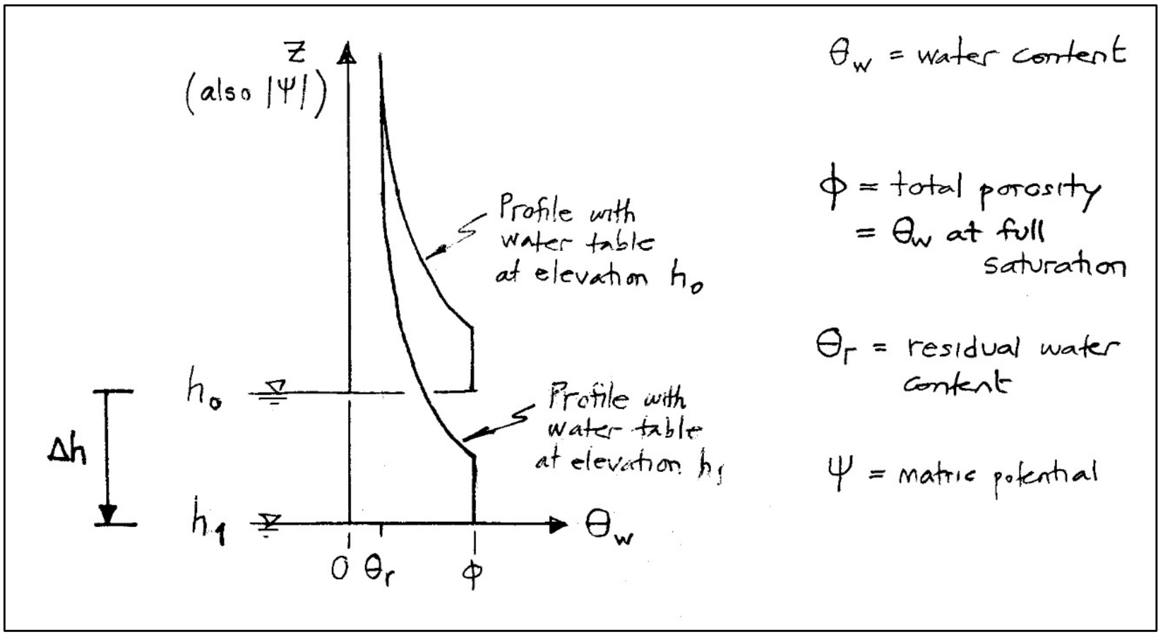
For pumping tests conducted in unconfined aquifers, the effective storage coefficient is the *specific yield*. The specific yield is defined as the volume of water that an unconfined aquifer releases from storage per unit surface area of aquifer per unit decline in the water table. The specific yield is the unconfined counterpart of the storativity and like storativity it is dimensionless.



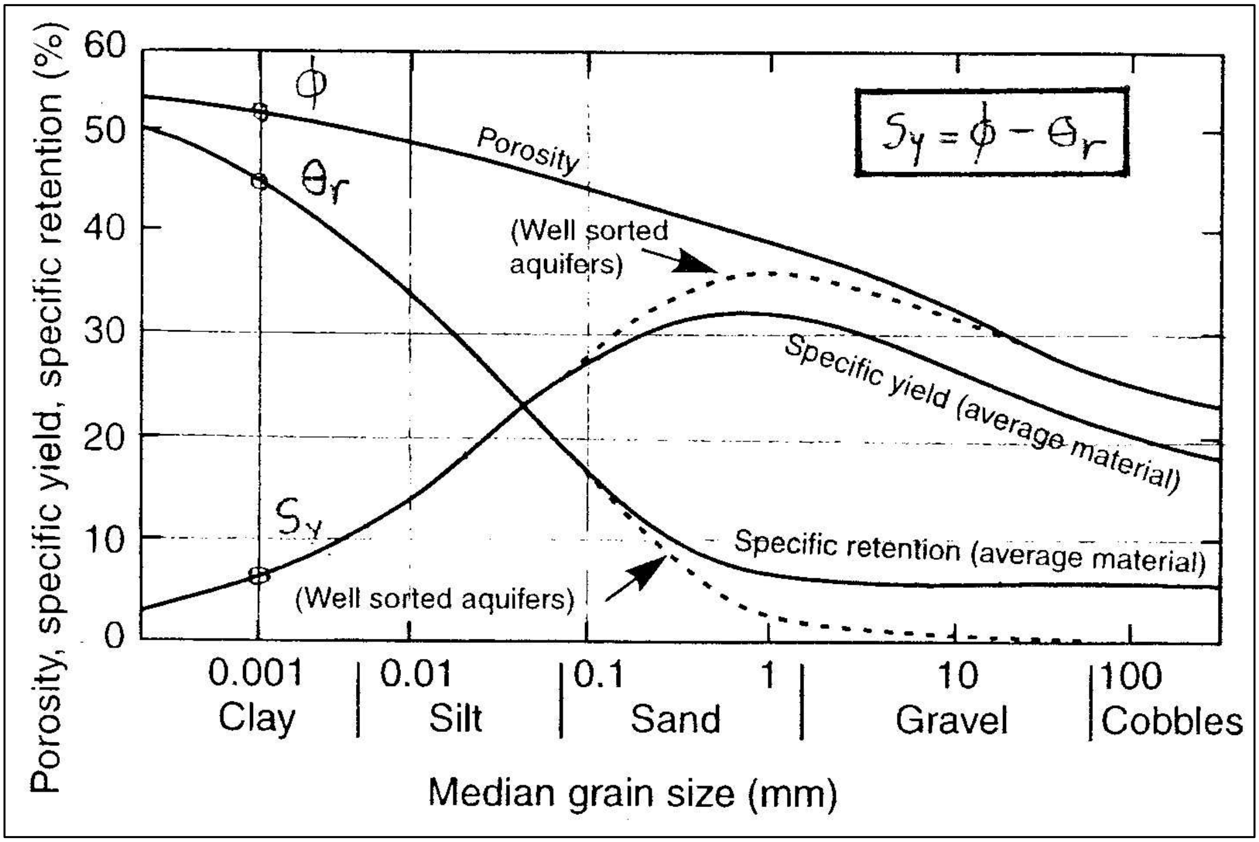
The specific yield corresponds to the difference between the saturated water content (porosity), ϕ, and the residual water content, θ_r:

$$S_y = \phi - \theta_r$$

The specific yield is also referred to as the *drainable porosity* and in some references as an *effective porosity*.



Relations between the specific yield and the soil characteristics, expressed as median grain size, are presented in several sources. These relations can be used to obtain first-cut estimates of the specific yield. The relations plotted in Stephens and others (1988) are reproduced here. Other versions of this figure are presented in Johnson (1967), Davis and DeWiest (1966) and Davis (1969). For sand and gravel aquifers, the long-term specific yield varies between about 20 and 30%.



Source: Stephens and others (1988)